Preprint YerPhI-1513(13)-98

ON THE THEORY OF RELATIVISTIC STRONG PLASMA WAVES

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Abstract

The influence of motion of ions and electron temperature on nonlinear one-dimensional plasma waves with velocity close to the speed of light in vacuum is investigated. It is shown that although the wavebreaking field weakly depends on mass of ions, the nonlinear relativistic wavelength essentially changes. The nonlinearity leads to the increase of the strong plasma wavelength, while the motion of ions leads to the decrease of the wavelength. Both hydrodynamic approach and kinetic one, based on Vlasov-Poisson equations, are used to investigate the relativistic strong plasma waves in a warm plasma. The existence of relativistic solitons in a thermal plasma is predicted.

PACS numbers: 52.35.Mw, 52.35.Fp

I. INTRODUCTION

Strong plasma waves passing through a plasma with phase velocity slightly smaller than the velocity of light are the subject of great interest during last two decades. Such waves can be excited in plasma by relativistic bunches of charged particles or laser pulses. The excited plasma waves can be used both to accelerate charged particles and to focus charged bunches [1]. Plasma-based accelerator concepts are currently under intensive development (see overview in Ref. [2] and numerous references therein). Accelerating gradients in the plasma wave can reach the values of tens GeV/m (notice that in conventional linacs accelerating gradients are in order of tens MeV/m), that is confirmed in recent experiments [2]. Also the focusing fields can be much more than that reached in conventional focusing magnetic systems. The acceleration of charged particles by relativistic strong waves also is considering as a possible mechanism of ultra-high energy (up to 10^{20} eV) cosmic ray generation in astrophysical plasma.

In a cold plasma an amplitude of one-dimensional plasma wave is limited by the wavebreaking field. In nonrelativistic case, when the wave phase velocity v_{ph} is much less than velocity of light ($v_{ph} \ll c$), the wavebreaking amplitude is equal to [3] $E_* = m_e \omega_{pe} v_{ph}/|e|$, where $\omega_{pe} = (4\pi n_0 e^2/m_e)^{1/2}$ is the electron plasma frequency, n_0 is the density of electrons in unperturbed plasma, m_e and e are the electron rest mass and their charge; the ions assumed to be immobile. In relativistic case the wavebreaking field is equal to [4] $E_{rel} = [2(\gamma - 1)]^{1/2}/\beta$, here $\beta = v_{ph}/c$, $\gamma = (1 - \beta^2)^{-1/2}$ is the relativistic factor and E_{rel} is normalized on E_*. The one-dimensional relativistic strong waves (RSW) can be excited in a plasma by wide relativistic bunches of charged particles or intensive laser pulses [2] (when $k_p a \gg 1$, where $k_p = \omega_{pe}/v_{ph}$, a is the characteristic transverse sizes of bunches or pulses).

Another important characteristic of nonlinear plasma waves is the dependence of the wavelength on the wave amplitude. Both in linear case and in nonlinear nonrelativistic one, the plasma wavelength in cold plasma is $\lambda_p = 2\pi v_{ph}/\omega_{pe}$. In the relativistic nonlinear regime, when plasma electrons get a relativistic velocity in the process of oscillations, nonlinear wavelength increases with the amplitude [5-8]. In the ultra-relativistic case ($\gamma \gg 1$), for the wave amplitude $E_{mp} \cong E_{rel}$, the wavelength approximately is equal to [5] $4(2\gamma)^{1/2}\lambda_p$. One can see that for large γ the wavelength is essentially more

than usual linear plasma wavelength.

In the previous studies the plasma ions, in the process of oscillations, including the nonlinear relativistic waves, usually were assumed to be immobile due to their large mass. In Ref. [9] it is shown that when $\gamma \ll (M/16m_e)^{1/3}$ (here M is mass of an ion; for example, for hydrogen plasma, consisting of protons and electrons, this condition gives $\gamma < 5$), the wavebreaking amplitude approximately is equal to E_{rel} and the motion of ions can be neglected. However, the dispersion properties of the relativistic strong plasma waves, which take into consideration the motion of ions, are not elucidated up to now. This problem has been considered in Sec. II on the base of cold hydrodynamics equations for the arbitrary γ and mass of particles forming the plasma. The necessity to take into account the ion motion conditioned by the following reasons. Firstly, because the maximum relativistic wavelength and amplitude grow proportional to $\gamma^{1/2}$, the plasma ions (even heavy ions) in such strong field can get velocity which is sufficient to give essential contribution in the process of charge separation in the wave. On the other hand, in a semiconductor plasma positively-charged particles (holes) have a mass similar or less than that of electron. The problem has also astrophysical aspect. The pole region of the pulsars is considered to be filled with an electron-positron plasma in which the strong plasma waves can be excited and high energy charged particles generated [10]. It is obvious that in the plasma wave passing through an electron-positron plasma, neither electrons nor positrons may be considered as neutralizing background.

Another important problem is the influence of plasma temperature on RSW. Finite plasma temperature has decisive significance for description of RSW near "wavebreaking". Actually, according to one-dimensional theory of relativistic plasma waves in a cold plasma, at the "breaking" point hydrodynamic plasma electron velocity is equal to the phase velocity and the density of electrons n_e tends to infinity [5-7, 11] (in this case spatial behavior of the density is similar to the δ-function). On the other hand, in a thermal plasma (even when the temperature is low), the pressure tends to infinity when $n_e \to \infty$. Thus, in this case, the pressure, as well as the plasma temperature, should be taken into consideration. In Ref. [12] the finite plasma temperature effect on nonrelativistic ($v_{ph} \ll c$) wavebreaking field is considered using 1D waterbag model for the distribution function. In this model it is assumed, that the electron distribution function during oscillations is constant in a limited interval of velocities and is equal to zero outside of

this interval. It is shown [12] that maximum amplitude of the plasma waves decreases with the temperature. In Ref. [13] the 1D relativistic waterbag model is used to investigate RSW in a warm plasma. Using relativistic equation of motion with the pressure term for plasma electrons, in Ref. [11] it is shown that in the case $\gamma \gg m_e v_{ph}^2/3T$ (where T is the temperature of electrons) the wavebreaking field is proportional to $T^{-1/4}$. The authors of Ref. [14] have analyzed the influence of low temperature ($T \ll m_e c^2$) on excitation of nonlinear wake fields by a relativistic charged bunches. They considered the equation of motion obtained using second moments of the distribution function. In the present paper (Sec. III) the hydrodynamics equations are used to study dispersion properties of RSW in a warm plasma. The dispersion correlation for weakly nonlinear case is obtained. In Sec. IV the strong plasma waves are investigated on the base of relativistic Vlasov kinetic equation and the Poisson equation.

II. ION MOTION EFFECT ON DISPERSION PROPERTIES OF RELATIVISTIC STRONG PLASMA WAVES

In this section we consider a cold uniform plasma consisting of positively-charged particles (for example, protons or positrons) with mass m_+ and electric charge q_+, and negatively-charged particles (electrons or negatively-charged ions) with mass m_- and the charge q_-. The relativistic equation of motion and the continuity equation for each plasma component and the Poisson equation for one-dimensional steady plasma waves are:

$$(\beta - \beta_\pm)\frac{d(\beta_\pm \gamma_\pm)}{dz} = -\frac{q_\pm}{|q_-|}\beta^2 E, \tag{1}$$

$$\beta \frac{dN_\pm}{dz} - \frac{d(N_\pm \beta_\pm)}{dz} = 0, \tag{2}$$

$$\frac{dE}{dz} = 1 - N_- + |q_+/q_-| N_+, \tag{3}$$

where $z = k_p(Z - v_{ph}t)$, $k_p = \omega_p/v_{ph}$, $\omega_p = (4\pi n_{0-} q_-^2/m_-)^{1/2}$, n_{0-} is the density of negatively-charged particles in equilibrium, $\beta_\pm = v_\pm/c$ are dimensionless velocities, $\gamma_\pm = (1 - \beta_\pm^2)^{-1/2}$, densities $N_\pm$ are normalized on the equilibrium values. The electric field strength is normalized on the nonrelativistic wavebreaking field $m_- \omega_p v_{ph}/|q_-|$ and obeys the formula

$$E(z) = -(1/\beta^2)d\Phi/dz, \tag{4}$$

where $\Phi \equiv \Phi_- = 1 + |q_-|\varphi/m_-c^2 \geq 1/\gamma$, φ is the electric potential. From expressions (1), (2) and (4) we have:

$$\beta_\pm = [\beta - (\Phi_\pm^2 - \gamma^{-2})^{1/2}]/(\beta^2 + \Phi_\pm^2), \tag{5}$$

$$N_\pm = \beta\gamma^2[\Phi_\pm/(\Phi_\pm^2 - \gamma^{-2})^{1/2} - \beta]. \tag{6}$$

Substituting $N_\pm(\Phi_\pm)$ and expression (4) in the Poisson equation (3) one obtain the following differential equation of the second order for Φ:

$$\frac{d^2\Phi}{dz^2} + \beta^3\gamma^2[\frac{\Phi_+}{(\Phi_+^2 - \gamma^{-2})^{1/2}} - \frac{\Phi}{(\Phi^2 - \gamma^{-2})^{1/2}}] = 0, \tag{7}$$

Here $\Phi_+ = 1 - q_+\varphi/m_+c^2 = 1 + \mu(1 - \Phi) \geq 1/\gamma$ and $\mu = |q_+/q_-|m_-/m_+$. The electric potential φ is assumed to be equal to zero when plasma density is equal to the equilibrium density.

Equation (7) can be rewritten in the form

$$\begin{aligned} \frac{d^2\Phi}{dz^2} + \frac{dU}{d\Phi} &= 0, \qquad (8) \\ U &= \beta^3\gamma^2\{[\beta - (\Phi_+^2 - \gamma^{-2})^{1/2}]/\mu + [\beta - (\Phi^2 - \gamma^{-2})^{1/2}]\}. \end{aligned}$$

Here, for convenience, $U(\Phi)$ is chosen to be equal to zero in a point $\Phi = 1$, where it reaches a minimum. When $\mu \to 0$, Eq. (7) reduces to the known equation for nonlinear waves in a plasma with immobile ions [2,7]. Formally, Eq. (8) describes one-dimensional motion of a particle in a field with potential $U(\Phi)$; the values Φ and E correspond to the coordinate and velocity of this fictitious particle respectively. Function U determines the characteristic of the field moving through the plasma. In Fig. 1 this function is presented for $\gamma = 10$ ($\beta \approx 0.995$) and different values of μ. One can see that for the arbitrary parameters the solutions of Eq. (8) [or Eq. (7)] are the periodic plasma waves (including the wave with zero amplitude-unperturbed plasma). Integrating Eq. (8) we have

$$\frac{d\Phi}{dz} = -\beta^2 E = \pm[2(U_{\max} - U)]^{1/2}, \tag{9}$$

where $U_{\max}$ is maximum value of $U(\Phi)$ in the process of oscillations. From (9) it follows that the plasma wave amplitude is equal to $E_{mp} = (2U_{\max})^{1/2}/\beta^2$. Substituting maximum permissible value of $U(\Phi)$, which reaching at $\Phi = 1/\gamma$, in this expression, we find the wavebreaking field:

$$\begin{aligned} E_{WB} &= 2^{1/2}\gamma[1 + (1 - \xi_1^{1/2}\xi_2^{1/2})/\mu], \\ \xi_1 &= 1 + \mu,\ \xi_2 = 1 + \mu(\gamma - 1)/(\gamma + 1). \end{aligned} \tag{10}$$

In the case $\mu \ll 1$, from (10) follows the expression

$$E_{WB} \approx (1 + \mu/8)[2(\gamma - 1)]^{1/2}/\beta, \tag{11}$$

which reduces to the well known relativistic wavebreaking field for plasma with immobile ions, when $\mu = 0$ [4]. Fig. 2 shows the wavebreaking field E_{WB} depending on μ (for example, for electron-positron plasma $\mu = m_e/m_{pos} = 1$, for the hydrogen plasma $\mu = m_e/m_{prot} \approx 5.455 \times 10^{-4}$) for different values of γ. Both in nonrelativistic case and in relativistic one the wavebreaking field weakly increases with μ. For example, according to (10), in nonrelativistic case ($\gamma \approx 1$), $E_{WB}(\mu = 1)$ only $2(1 - 2^{-1/2})^{1/2} \approx 1.08$ times exceeds the wavebreaking amplitude at $\mu = 0$. Proceeding from the shape of the "potential" $U(\Phi)$ (see Fig. 1) one can expect that the plasma wavelength undergoes considerable change with μ. In Fig. 3 the dependence of relativistic plasma wavelength Λ_p [note, that according to the variables accepted in (1)-(3), the linear plasma wave at $\mu = 0$ corresponds to the value $\Lambda_p = 2\pi$] on amplitude presented. The curve 1 corresponds to the case of immobile ions and coincides with that previously obtained [7]; in this case Λ_p grows with the amplitude due to relativistic velocities of the oscillating plasma particles (notice, that in nonlinear nonrelativistic regime the plasma wavelength does not depend on amplitude). The motion of positive ions for fixed amplitude causes the decrease of charge separation length and therefore, leads to the decrease of the wavelength. Fig. 3 clearly shows competition of two tendencies. For the small μ the wavelength grows with the amplitude due to nonlinearity. With the increase of E_{mp}, the effect of ion motion becomes more essential. When μ is not small, the behavior of the wavelength caused mainly by ion motion. In electron-positron plasma ($\mu = 1$), Λ_p monotonously decreases with the increase of E_{mp}, in contrast to the case of heavy ions ($\mu \approx 0$). The results of simulations presented in Fig. 3 conform with the well known result of

linear theory ($E_{mp} \ll 1$; see, e.g., Ref. [15]): $\Lambda_p = \Lambda_{p0}/(1+\mu)^{1/2}$, where $\Lambda_{p0} = \Lambda_p(\mu = 0)$. Note also that the results practically did not change for arbitrary $\gamma \gg 1$. Considerable decrease of the relativistic plasma wavelength with μ is demonstrated in Fig. 4.

Previous studies have shown that energy of electrons (or positrons) accelerated in the field of relativistic nonlinear wave, in a cold plasma with immobile ions, can reach a value of $4m_ec^2\gamma^3$ [7,16]. In the general case the relativistic factor of a resonant electron passing from a point with the dimensionless potential Φ_1 to a point with Φ_2 is equal to [7]

$$\gamma_{acc} \approx \gamma_{acc}(0) + 2\gamma^2(\Phi_2 - \Phi_1), \tag{12}$$

where $\gamma_{acc}(0) \approx \gamma$ is value of the relativistic factor at the initial point. When $\mu = 0$, $\Phi_{\min} = 1/\gamma \leq \Phi \leq \Phi_{\max} \approx 2\gamma$ [7]. Substituting this maximum and minimum values in (12), for maximum energy of accelerated electrons one obtains $(\gamma_{acc})_{\max} \approx 4\gamma^3$. With μ growth, the maximum energy decreases due to the decrease of $\Phi_{\max}$ (see Fig. 1). For the case of electron-positron plasma function $U(\Phi)$ is symmetric with reference to axis $\Phi = 1$ and, as it is easy to see, in this case $\Phi_{\min} = 1/\gamma$, $\Phi_{\max} = 2 - 1/\gamma$. Then, the maximum energy of accelerated particles is $(\gamma_{acc})_{\max} \approx 4\gamma^2$, that is in γ times less than that in the case $\mu = 0$.

If a bunch of charged particles with density $n_b(z)$ and electric charge q_b passes through a plasma, adding to the left side of equation (7) the value $\alpha(z) = \beta^2(q_b/|q_-|)n_b(z)/n_{0-}$, we obtain the equation that describes the excitation of steady plasma wake fields by the bunch. In this case the phase velocity is equal to velocity of the bunch. In this section the properties of RSW have been investigated by simulation of wake wave generation by charged bunches.

Above we have considered the case $\mu \leq 1$. However, one can see that the obtained results are valid also for $\mu > 1$, if E we replace by $-E$ [this new E is normalized to $m_+\omega_p v_{ph}/q_+$, $\omega_p = (4\pi n_{0+}q_+^2/m_+)^{1/2}$], μ replace by $1/\mu$, replace the subscript " $+$ " by " $-$ " and vice versa.

III. THE INFLUENCE OF ELECTRON TEMPERATURE ON RELATIVISTIC NONLINEAR PLASMA WAVES: HYDRODYNAMIC APPROACH

Here we continue to consider the dispersion properties of RSW in the frame of hydrodynamic approach, investigate relativistic nonlinear waves in a

warm plasma. Adding the relativistic pressure term $-(\gamma_e^2/N_e)(1-\beta\beta_e)dP/dz$ [11,14] with $P=\tau(N_e/\gamma_e)^3$ [11] (which is relativistic generalization of usual equation of state for one-dimensional adiabatic compression) to the equation of motion of plasma electrons one can obtain the equations:

$$
\begin{aligned}
(\beta-\beta_e)\frac{d(\beta_e\gamma_e)}{dz} &= \beta^2 E + 3\beta^2\tau\frac{\gamma_e(1-\beta\beta_e)^2}{(\beta-\beta_e)^3}\frac{d\beta_e}{dz}, \\
\frac{dE}{dz} &= -\frac{1}{\beta^2}\frac{d^2\Phi}{dz^2} = 1 - N_e.
\end{aligned} \tag{13}
$$

In (13) $\beta_e = v_e/c$ and $\tau = T/m_e c^2$ are dimensionless velocity and temperature of plasma electrons, $\gamma_e = (1-\beta_e^2)^{-1/2}$, $\Phi = 1 + |e|\varphi/m_e c^2$. The plasma ions are assumed to be immobile due to their large mass. The density of electrons $N_e = n_e/n_0$ normalized to the unperturbed value n_0, as usually, is obtained from the continuity equation:

$$N_e = \beta/(\beta-\beta_e). \tag{14}$$

When $\tau \to 0$ equations (13) and (14) describe RSW in a cold plasma [5,6]. Note, that the value $\tau = 1$ corresponds to the temperature of about 6×10^9 K. For laboratory plasmas the temperature changes in the bounds $\tau \sim 10^{-6} \div 10^{-2}$; for star plasmas $\tau \sim 10^{-5} - 1$.

The dispersion correlation can be obtained analytically for weakly nonlinear wave, when $u = \beta_e(z)/\beta \ll 1$. In this case from equations (13) and (14) we have:

$$(a_0 - a_1 u + a_2 u^2)\frac{d^2u}{dz^2} - (a_1 - 2a_2 u)\left(\frac{du}{dz}\right)^2 + u + u^2 + u^3 = 0, \tag{15}$$

$a_0 = 1-3\tau/\beta^2, a_1 = 1+3\tau(3-2\beta^2)/\beta^2, a_2 = 3\beta^2/2-3\tau(6-11\beta^2/2+\beta^4)/\beta^2.$

Looking for solution of Eq. (15) as (see, e.g., Ref. [17])

$$
\begin{aligned}
u &= \varepsilon u_1(\Psi) + \varepsilon^2 u_2(\Psi) + \varepsilon^3 u_3(\Psi) + ..., \\
d\Psi/dz &= \lambda_p/\Lambda_p = \kappa_0 + \varepsilon\kappa_1 + \varepsilon^2\kappa_2 + ...,
\end{aligned}
$$

we obtain

$$\begin{aligned}\lambda_p/\Lambda_p &= a_0^{-1/2}(1+b\beta_m^2), \\ b &= -3/16+3\tau a_0^{-1}(10-9\beta^2+\beta^4)/8\beta^4+3\tau^2 a_0^{-2}(2-\beta^2)^2/\beta^6,\end{aligned} \tag{16}$$

where $\varepsilon = \beta_m/\beta \ll 1$ is the small parameter, $\beta_m = (\beta_e)_{\max}$ and Λ_p is the wavelength. In the linear case ($\beta_m^2 \to 0$) from (16) it follows that the wavelength decreases with the temperature according to well known Bohm-Gross dispersion correlation: $\Lambda_p = \lambda_p(1-3\tau/\beta^2)^{1/2}$. On the other hand, in a cold plasma ($\tau = 0$) the wavelength increases due to nonlinearity (see, e.g., Ref. [15]): $\Lambda_p \approx \lambda_p(1+3\beta_m^2/16)$. Fig. 5 shows the dependence of the wavelength on wave amplitude in thermal plasma obtained by simulation of Eqs. (13) and (14). In the case of low temperature this dependence almost coincides with that in a cold plasma (compare curves 1 in Figs. 5 and 3).

In the case $\beta \to 1$ equations (13) and (14) can be easily integrated (see also Ref. [11]):

$$\Phi - \left(\frac{1-\beta_e}{1+\beta_e}\right)^{1/2} - 3\tau\left[\left(\frac{1+\beta_e}{1-\beta_e}\right)^{1/2} - 1\right] = 0. \tag{17}$$

One can see that the thermal term can not be neglected near the "wavebreaking" ($\beta_e \to 1$) even for low temperature. In the latter case the wavebreaking field is proportional to $\tau^{-1/4}$ [11].

In the frame of hydrodynamic theory the velocity of plasma electrons can not exceed the wave phase velocity. Actually, if $\beta_e > \beta$, then, according to expression (14), the density of electrons becomes negative, that has no physical sense. In reality, when $\beta_e \approx \beta$ in a warm plasma (even when the temperature is low), considerable part of electrons gets velocities more than the phase velocity due to their thermal energy distribution. When the temperature is not low, the energy distribution effect on plasma waves is essential in all cases. Therefore, the strong waves near the "wavebreaking" and when τ is not low, can be described correctly in the frame of kinetic approach.

IV. KINETIC THEORY OF THE RELATIVISTIC STRONG PLASMA WAVES

As in the previous section, here we assume plasma ions to be immobile. The kinetic approach, for one-dimensional steady fields passing through a

warm plasma, gives the following system, obtained from relativistic Vlasov equation and Maxwell equations (see, e.g., Ref. [18]):

$$\left[\beta - \frac{p}{(1+p^2)^{1/2}}\right]\frac{\partial f}{\partial z} - \frac{\partial \Phi}{\partial z}\frac{\partial f}{\partial p} = 0, \tag{18}$$

$$\frac{d^2\Phi}{dz^2} + \beta^2(1 - N_e) = 0, \tag{19}$$

$$N_e = \int_{-\infty}^{+\infty} f(p,z)dp, \tag{20}$$

where $p = p_z$ is the plasma electron momentum, normalized to $m_e c$, $f(p,z)$ is the distribution function. In an unperturbed plasma the distribution function is equal to 1D relativistic Maxwell distribution [19]

$$\begin{aligned} f_0 &= A[1 + (1+p^2)^{1/2}/\tau]\exp[-(1+p^2)^{1/2}/\tau], \qquad (21)\\ A &= \tau/2K_2(1/\tau), \\ \langle p^2\rangle_0 &= \tau[K_1(1/\tau)/K_2(1/\tau) + 4\tau], \end{aligned}$$

$$\langle(1+p^2)^{1/2}\rangle_0 = 2\tau + (1-\tau^2)K_1(1/\tau)/K_2(1/\tau),$$

where $K_n(x)$ is the modified Bessel function of the n-th order. In (21) we also have written out average squared pulse and total energy for the one-dimensional equilibrium distribution f_0, that may be interesting for future investigations.

Solving Eq. (18) by the method of characteristics (see, e.g., Ref. [20]) and requiring that the function f reduces to the equilibrium distribution (21) at $\Phi = 1$, one obtains the following general solution:

$$\begin{aligned} f &= A(1 + S/\tau)\exp(-S/\tau), \qquad (22)\\ S &= (1+g^2)^{1/2},\ g = -\gamma^2[\beta r \pm (r^2 - \gamma^{-2})^{1/2}], \\ r &= \beta p - (1+p^2)^{1/2} + \Phi - 1, \end{aligned}$$

In the expression for g, the plus sign corresponds to the case $p \le \beta\gamma$ and the minus sign to $p > \beta\gamma$; in equilibrium ($\Phi = 1$) we have $g = p$. Substituting

the expressions (20) and (22) in Eq. (19), one obtains equation for Φ. The plasma electron density $N_e(\Phi)$ obtained numerically from (20) and (22) is presented in Fig. 6. In the case of low temperature, the integral in (20) can be calculated by the Laplace asymptotic method [19]. The value of N_e is at maximum when $\Phi \approx 1/\gamma$ and is equal to

$$N_{\max} \approx [\Gamma(1/4)/4]\gamma(\beta\gamma/\pi)^{1/2}(2/\tau)^{1/4} \approx 0.6\gamma(\beta\gamma)^{1/2}\tau^{-1/4}. \qquad (23)$$

According to expression (23), in a cold plasma $N_{\max} \to \infty$, that conforms with the previous investigations [5,6,11]. When $\tau \ll 1$ and $\Phi > 1/\gamma$, the dependence $N_e(\Phi)$ approximately is described by expression (6).

Simulations of the problem show that the plasma wavelength increases with the wave amplitude and tends to infinity for a solitary wave (soliton). Fig. 7 shows dependence of the maximum value of the amplitude (which corresponds to the maximum of electric field strength in the soliton E_s) on plasma temperature for different γ. One can see that E_s is almost constant and equal to the relativistic wavebreaking field E_{rel} for the values of τ up to $0.05 - 0.1$ and then decreases rapidly.

As it was mentioned above, the relativistic plasma waves can be excited by charged bunches or laser pulses. In order to describe the excitation of the wake field by a charged bunch, it is necessary to add in the left side of Eq. (19) the value $\beta^2\alpha(z)$ [the definition of $\alpha(z)$ see in Sec. II]. The nonlinear periodical wave and the solitary wave excited by uniform electron bunch are plotted in Fig. 8. The plasma electron density behind the soliton tends to its equilibrium value ($N_e \to 1$) and the electric field strength tends to zero. However, in this case the average plasma electron momentum $\langle p\rangle = \int_{-\infty}^{+\infty} pf(p,z)dp / \int_{-\infty}^{+\infty} f(p,z)dp$ tends to a non-zero constant value. This does not seem strange because the solitary wave can be considered as a wave with infinite wavelength. Hence, bulk motion of plasma electrons behind the solitary wave takes place, while the plasma remains neutral. Equations (18)-(21) have also non-periodical solutions. However, such solutions have no physical sense [19] and should be considered in the frame of non-stationary kinetic theory. Thus, in a thermal plasma with immobile ions two kinds of steady waves can exist: periodical waves and solitons.

V. CONCLUSIONS

The results presented in this paper supplement the theory of nonlinear relativistic plasma waves, taking into consideration motion of ions and finite

plasma temperature. It is shown that the nonlinearity leads to the increase of relativistic wavelength, while ion motion leads to the wavelength decrease. For example, in electron-positron plasma the wavelength monotonously decreases as the amplitude increases. The relativistic wavebreaking field weakly depends on the ion mass.

Contrary to the case of cold plasma, in a warm plasma the relativistic solitary waves (solitons) can exist. The plasma wavelength grows monotonously with the amplitude in the warm plasma due to nonlinearity. It has been found that maximum electron density in the plasma wave decreases with the temperature as $T^{-1/4}$ and tends to infinity in a cold plasma, that was shown by previous investigations.

ACKNOWLEDGMENTS

The author would like to thank S. S. Elbakian and E. V. Sekhpossian for helpful discussions and H. Nersisyan for the help in preparing the manuscript for publication.

This work was supported by the International Science and Technology Center, Project A-013.

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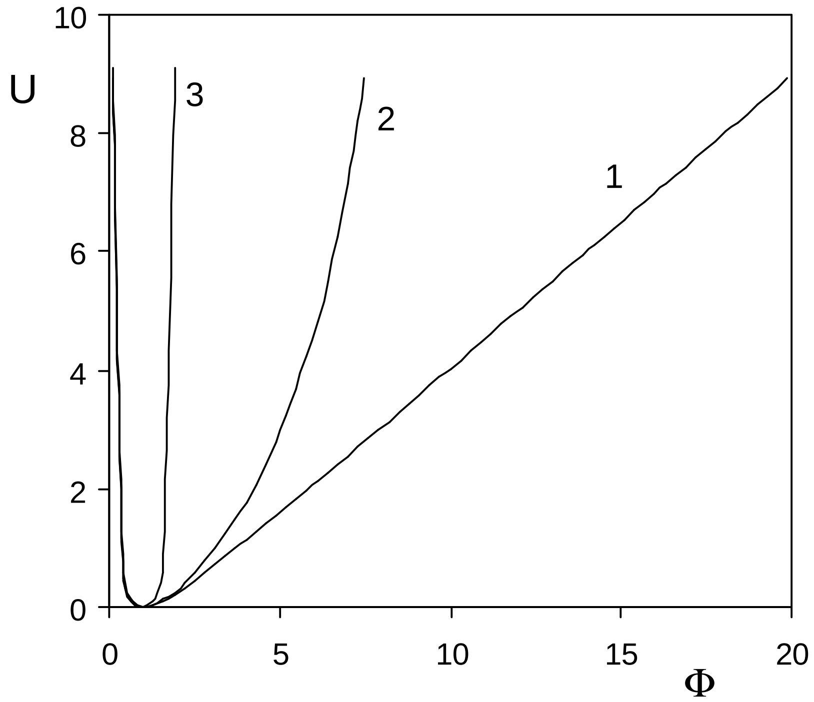

Figure 1. "Potential" $U(\Phi)$ for the value γ=10.

1- μ=0; 2- μ=0.1; 3- μ=1.

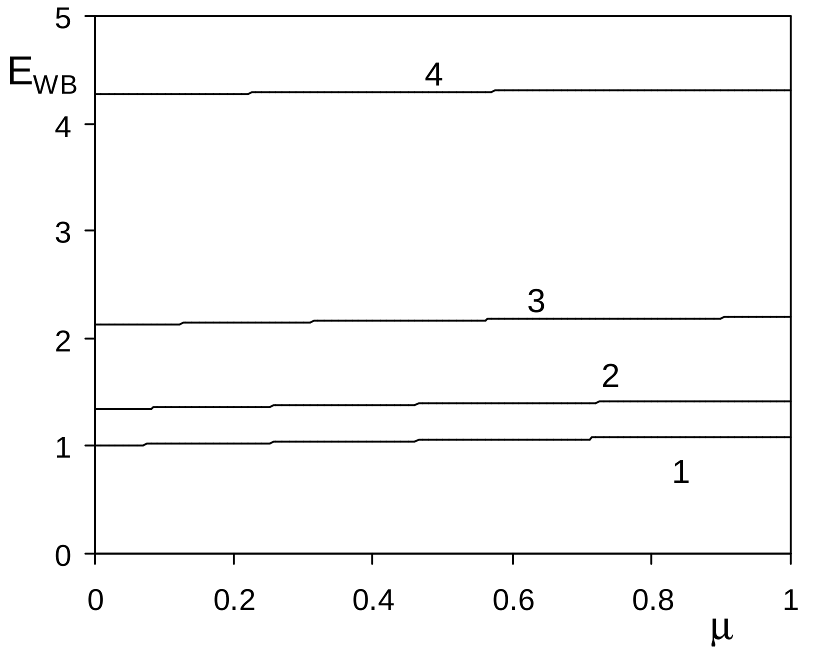

Figure 2. The wavebreaking field depending on μ.

1- γ=1.01; 2- γ=1.5; 3- γ=3; 4- γ=10.

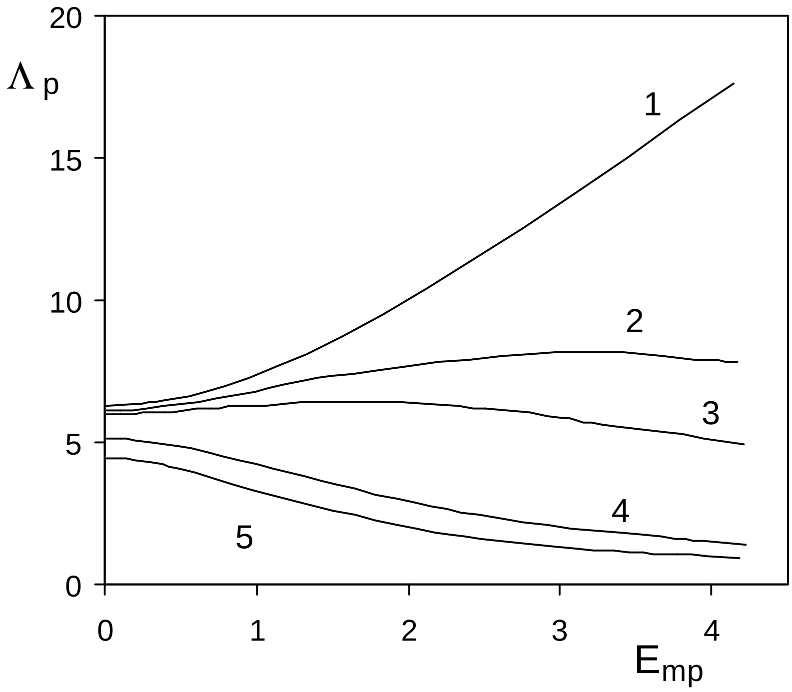

Figure 3. Relativistic plasma wavelength Λ_p depending on the electric field amplitude E_{mp} ; γ=10. 1- μ=0; 2- μ=0.05; 3- μ=0.1. 4- μ=0.5; 5- μ=1.

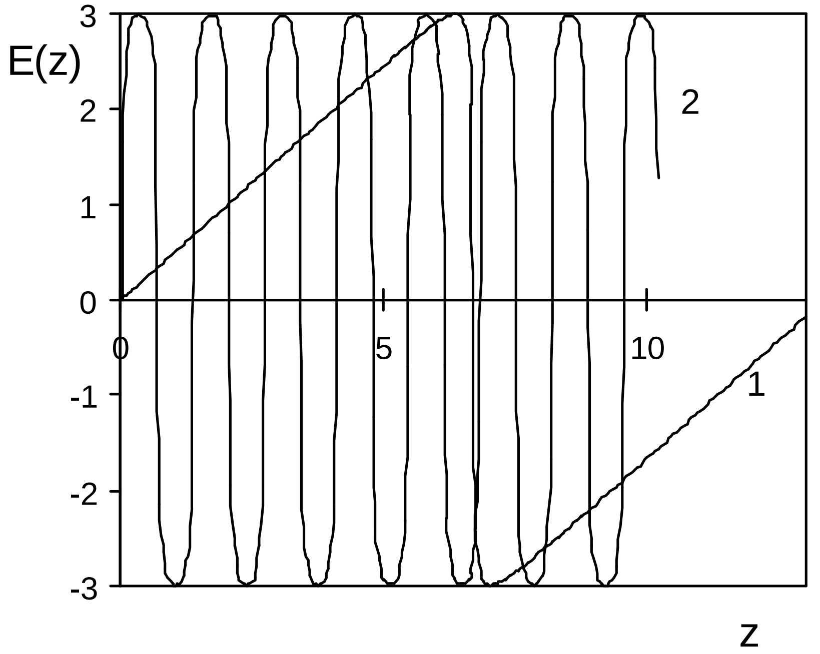

Figure 4. Relativistic strong wave in plasma with immobile ions (1- μ=0) and in electron-positron plasma (2- μ=1); γ=10.

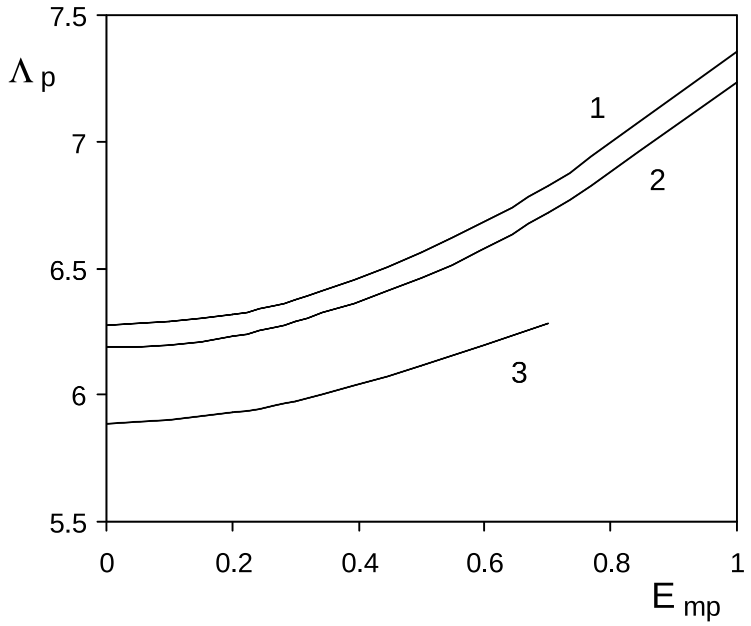

Figure 5. The relativistic plasma wavelength in a warm plasma as a function of the wave amplitude; γ=10. 1- τ=0.001; 2- τ=0.01; 3- τ=0.04.

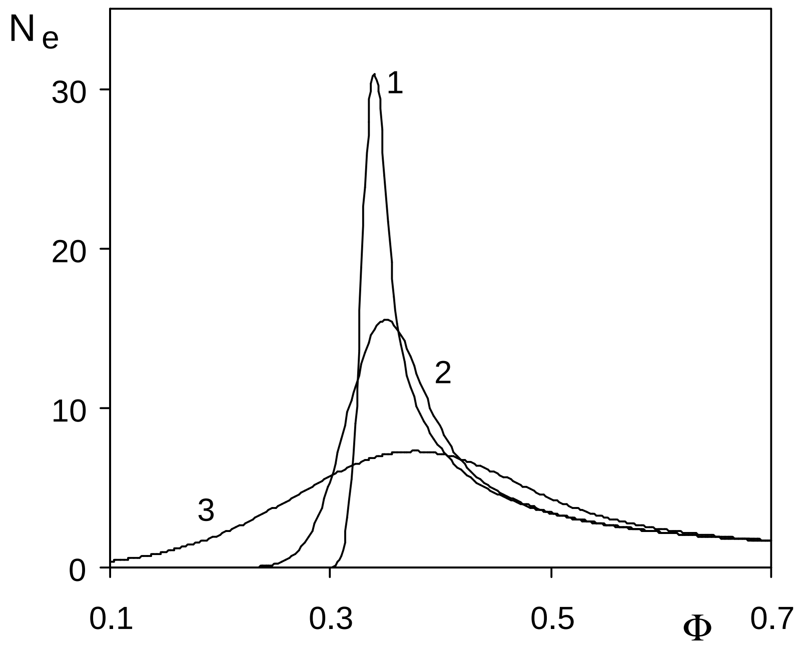

Figure 6. Plasma electron density as function of the dimensionless electric potential Φ for γ=3. 1- $\tau=10^{-4}$; 2- $\tau=10^{-3}$; 3- $\tau=10^{-2}$.

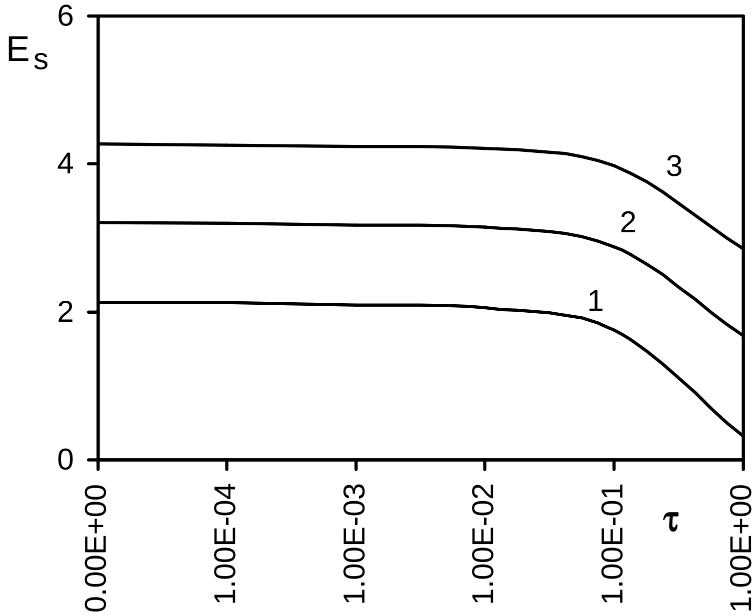

Figure 7. The maximum value of electric field amplitude of nonlinear plasma wave depending on plasma electron temperature.

1- γ=3; 2- γ=6; 3- γ=10.

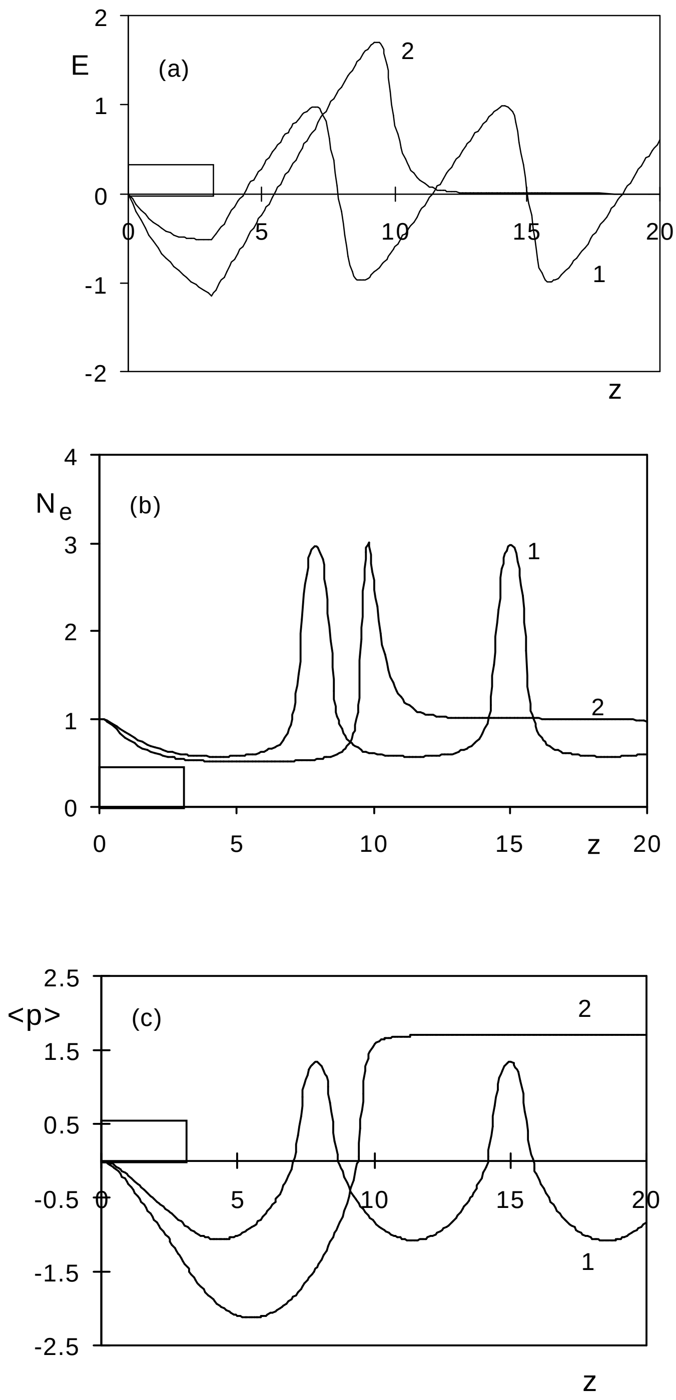

Figure 8. Electric field strength (a), plasma electron density (b), and average electron momentum (c) in strong plasma field, excited by uniform electron bunch (γ=3, τ=0.1). The rectangles show the bunch.
1- periodic wave, n_b/n_0=0.4; 2- solitary wave, n_b/n_0=0.6575.